

Cooperative control of autonomous vehicles: theory and applications

Nguyen T. Hung

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Institute for Systems and Robotics (ISR), Instituto Superior Tecnico (IST), University of Lisbon

VNCR seminar, July 2020



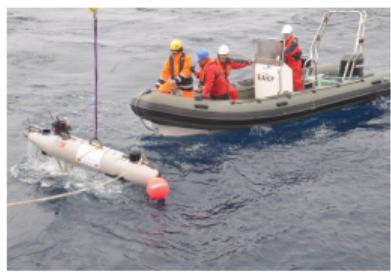
IST Lisbon and DSOR group



IST campus, viewed from Alameda park



DSOR group

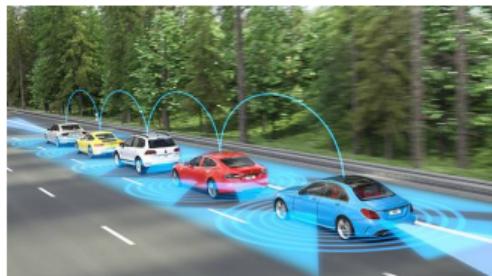
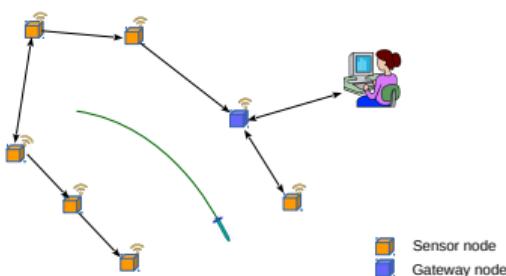


Multi agent systems (MAS) ▷ Examples

Nature behaviors



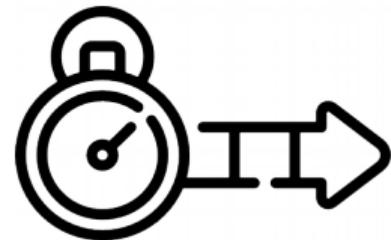
Engineering applications



What next?

- 1 A brief review
- 2 Cooperative of MAS with event-triggered communications
- 3 Applications in cooperative control of autonomous vehicles
- 4 Others applications

A brief review



Cooperative control of MAS ▷ A brief review

Vicsek et al. (1995)

VOLUME 75, NUMBER 6

PHYSICAL REVIEW LETTERS

7 AUGUST 1995

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,^{1,2} András Czirók,¹ Eshel Ben-Jacob,³ Inon Cohen,³ and Ofer Shochet³

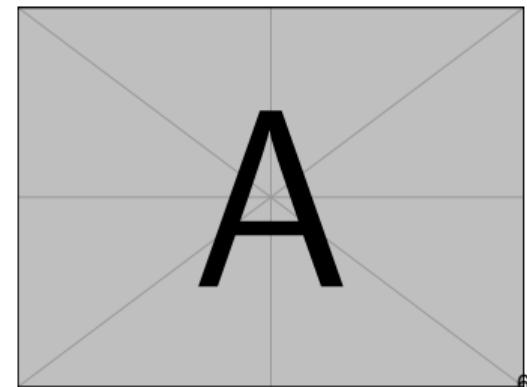
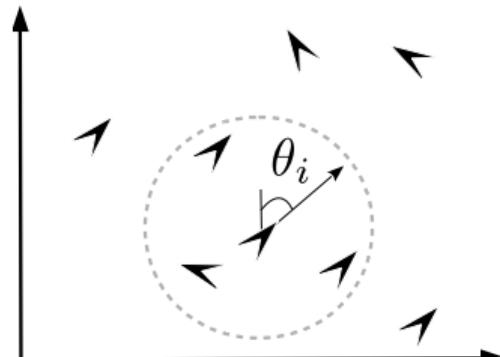
¹*Department of Atomic Physics, Eötvös University, Budapest, Puskin u 5-7, 1088 Hungary*

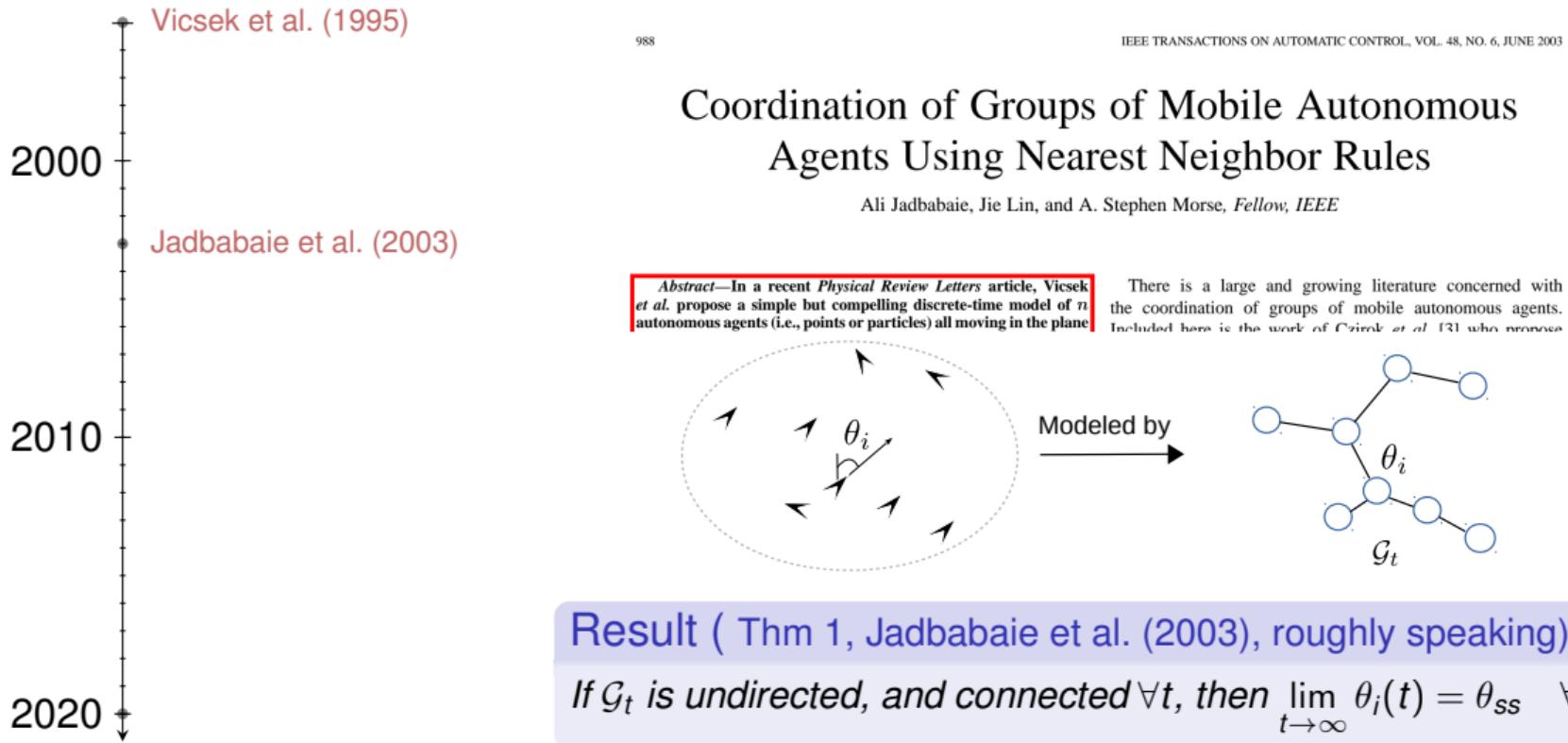
²*Institute for Technical Physics, Budapest, P.O.B. 76, 1325 Hungary*

³*School of Physics, Tel-Aviv University, 69978 Tel-Aviv, Israel*

(Received 25 April 1994)

$$\theta_i(t+1) = \frac{1}{1 + |\mathcal{N}_i(t)|} (\theta_i(t) + \sum_{j \in \mathcal{N}_i(t)} \theta_j(t))$$





Vicsek et al. (1995)

1520

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 9, SEPTEMBER 2004

2000

Jadbabaie et al. (2003)
Olfati-Saber & Murray (2004)

2010

2020

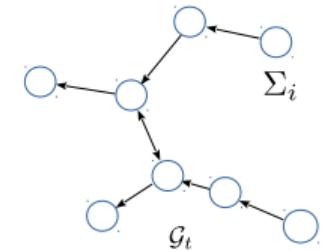
Consensus Problems in Networks of Agents With Switching Topology and Time-Delays

Reza Olfati-Saber, *Member, IEEE*, and Richard M. Murray, *Member, IEEE*

- Average consensus problem

$$\Sigma_i : \dot{x}_i = u_i, \quad i = 1, \dots, N$$

$$u_i = - \sum_{j \in \mathcal{N}_i(t)} (x_i - x_j).$$

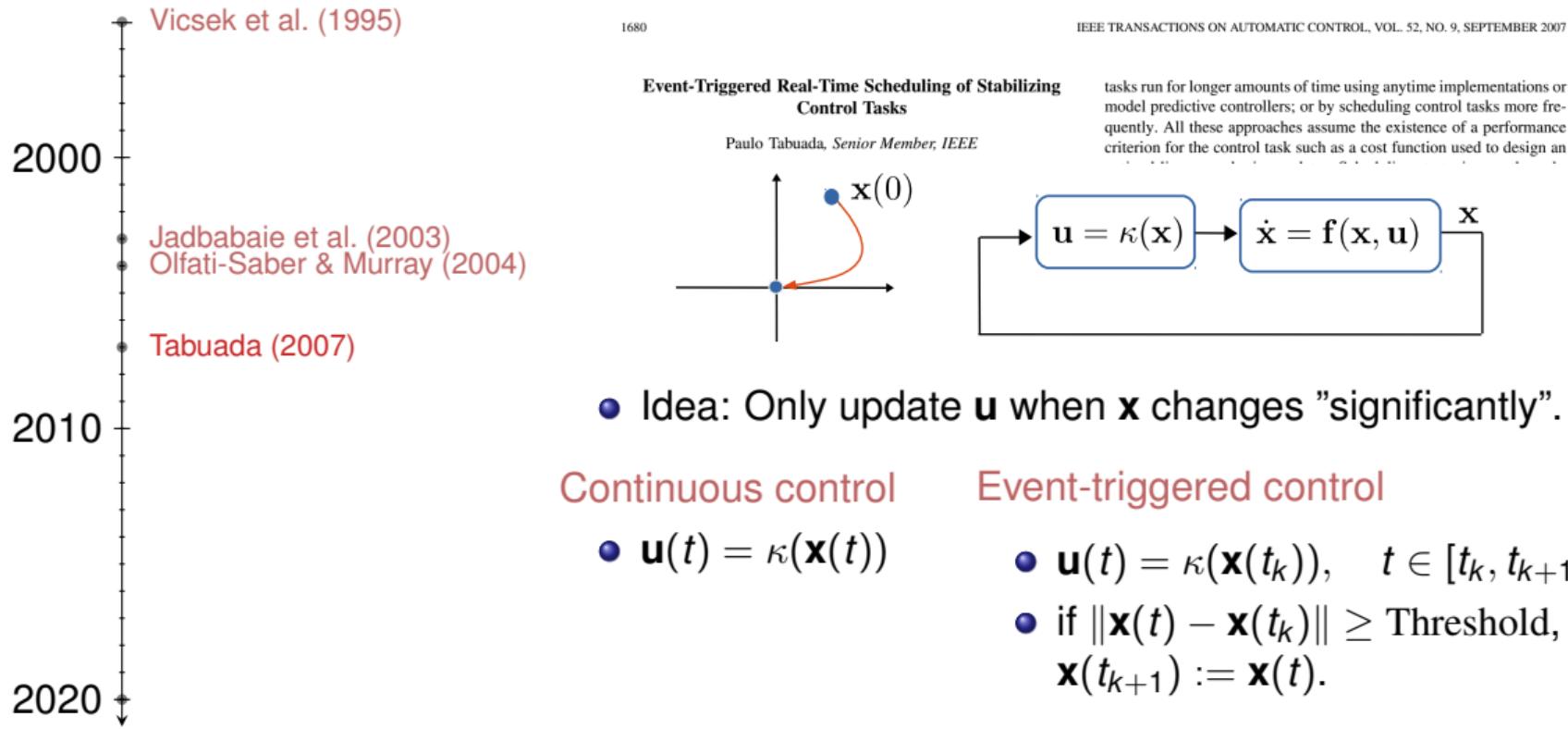


Result (Thm 9, Olfati-Saber & Murray (2004))

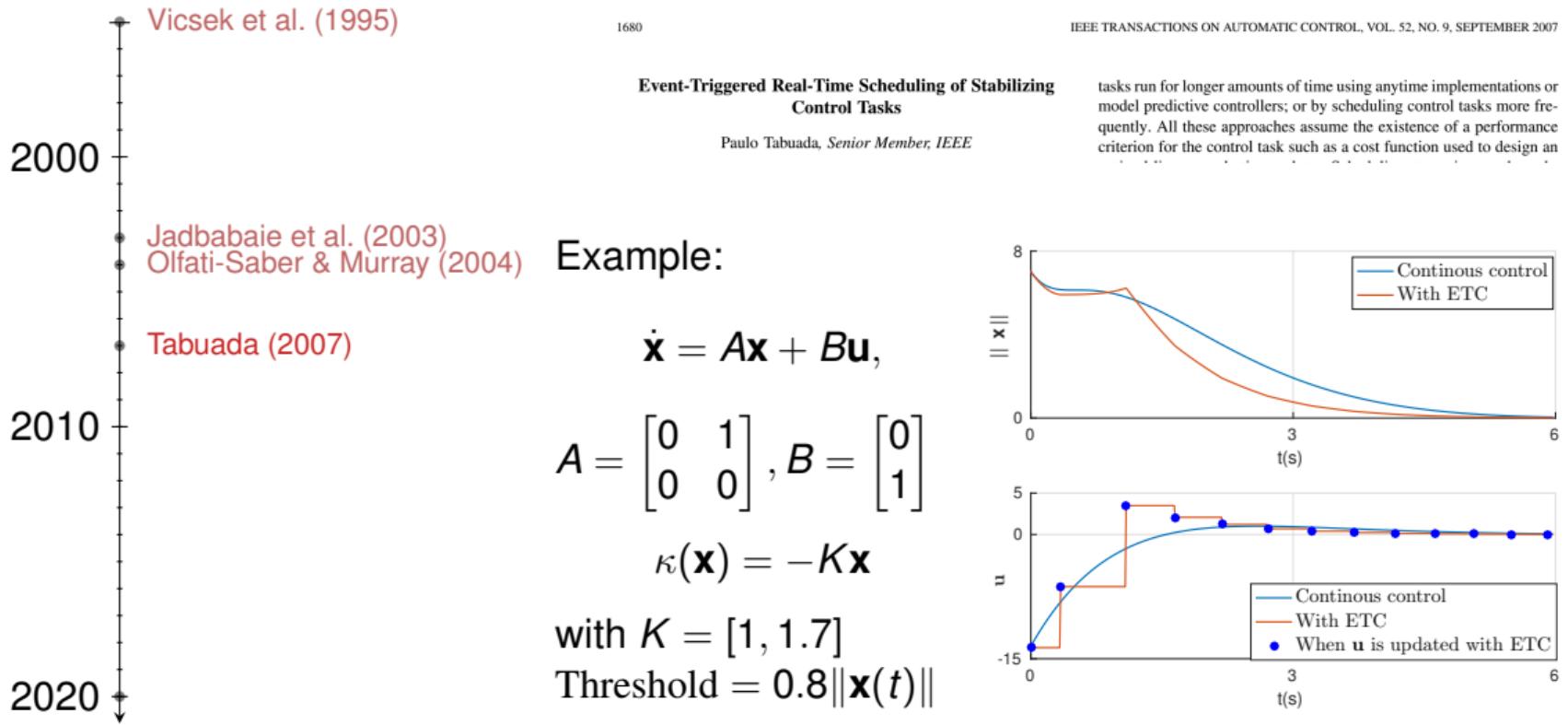
If \mathcal{G}_t is strongly connected and balanced $\forall t$, then

$$\lim_{t \rightarrow \infty} x_i(t) = \sum_{i=1}^N x_i(0)/N, \quad \forall i$$

Cooperative control of MAS ▷ A brief review



Cooperative control of MAS ▷ A brief review



Cooperative control of MAS ▷ A brief review

Vicsek et al. (1995)

51st IEEE Conference on Decision and Control
December 10-13, 2012. Maui, Hawaii, USA

2000

Jadbabaie et al. (2003)
Olfati-Saber & Murray (2004)

An Introduction to Event-triggered and Self-triggered Control

W.P.M.H. Heemels K.H. Johansson P. Tabuada

2010

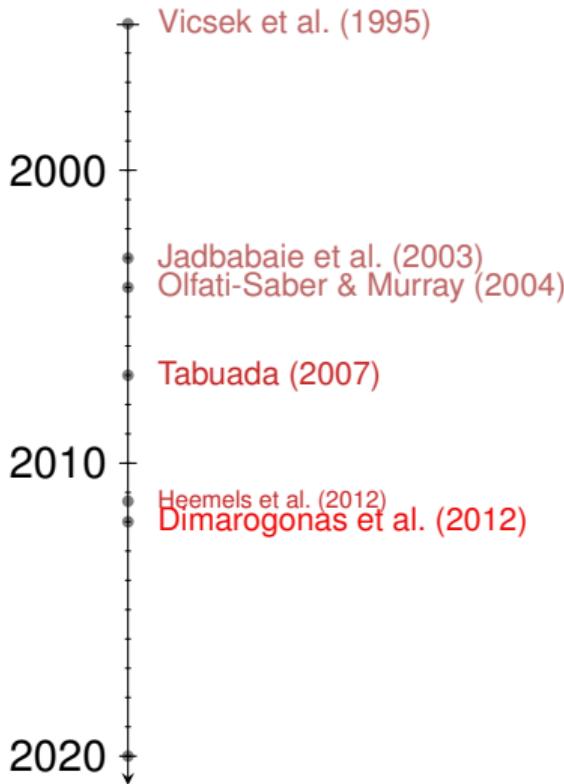
Tabuada (2007)

A nice tutorial about event triggered control and estimation!!!

Heemels et al. (2012)

2020

Cooperative control of MAS ▷ A brief review

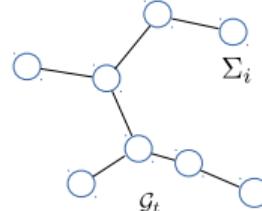


- [7] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: A survey of recent results," *IEEE Trans. Autom. Control*, vol. 54, no. 2, pp. 308–322, Feb. 2009.
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- [11] J. Zhao and D. J. Hill, "On stability, L_2 -gain and H_∞ control for switched systems," *Automatica*, vol. 44, pp. 1220–1232, 2008.
- [12] M. S. Branicki, "Multiple Lyapunov functions and other analysis tools

Distributed Event-Triggered Control for Multi-Agent Systems

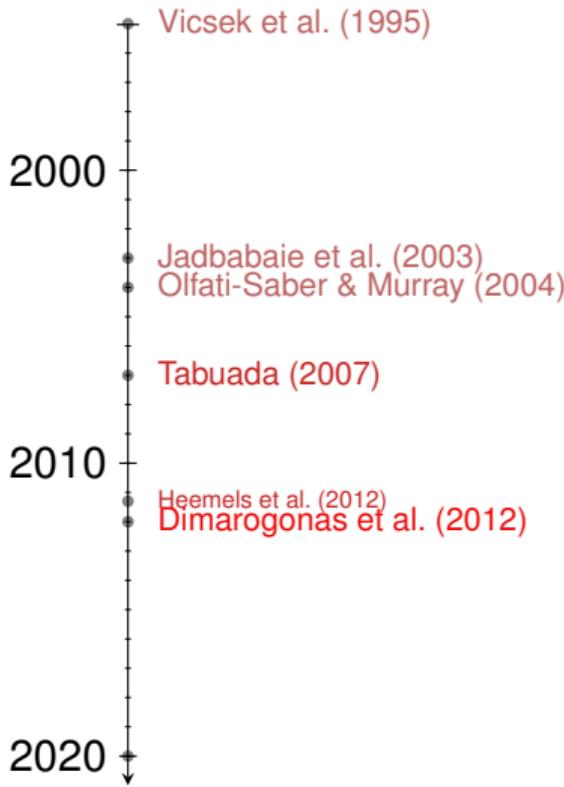
Dimos V. Dimarogonas, Emilio Frazzoli, and Karl H. Johansson

Abstract—Event-driven strategies for multi-agent systems are motivated by the future use of embedded microprocessors with limited resources that will gather information and actuate the individual agent controller updates. The controller updates considered here are event-driven, depending on the ratio of a certain measurement error with respect to the norm of a func-



- Apply idea in **Tabuada (2007)** for consensus problem
$$\Sigma_i : \quad \dot{x}_i = u_i, \quad i = 1, \dots, N.$$
- A mechanism to decide when u_i should be updated.

Cooperative control of MAS ▷ A brief review



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● Consensus control law

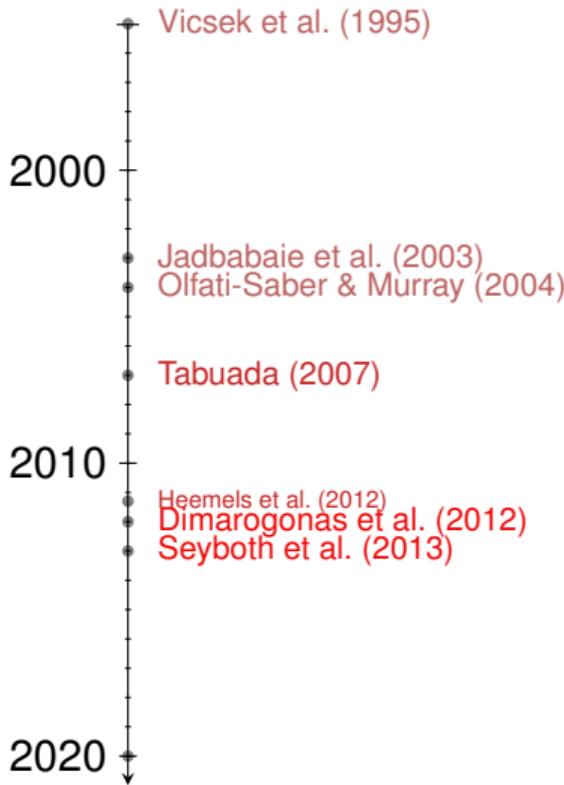
$$u_i = - \sum_{j \in \mathcal{N}_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

where $\hat{x}_i(t) = \hat{x}_i(t_{i,k}) \quad \forall t \in (t_{i,k}, t_{i,k+1})$

● update u_i when

$$|\hat{x}_i(t) - x_i(t)| \geq c \left(\sum_{j \in \mathcal{N}_i} x_i(t) - x_j(t) \right)^2$$

Cooperative control of MAS ▷ A brief review

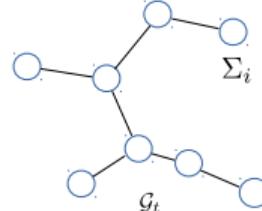


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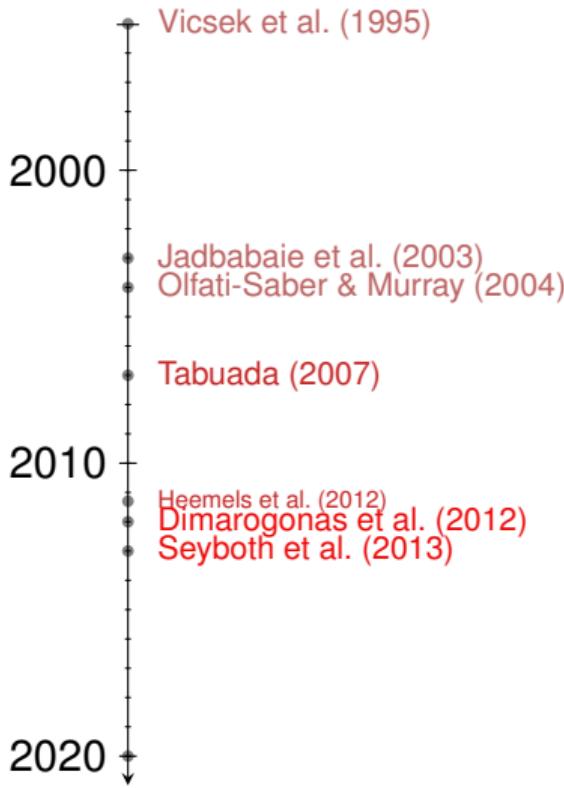
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- Apply idea in Tabuada (2007) for consensus problem
$$\Sigma_i : \quad \dot{x}_i = u_i, \quad i = 1, \dots, N.$$
- ETC decides when agents broadcast

Cooperative control of MAS ▷ A brief review



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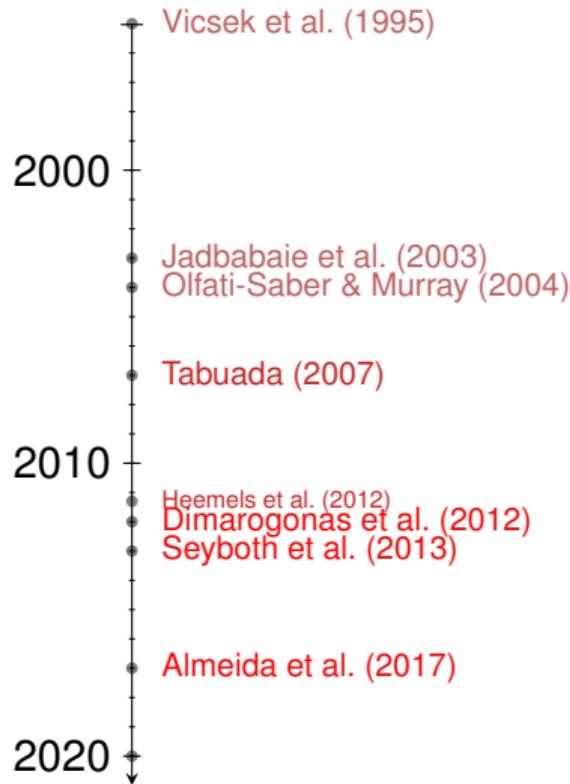
● Consensus control law

$$u_i = - \sum_{j \in \mathcal{N}_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

where $\hat{x}_i(t) = \hat{x}_i(t_{i,k}) \quad \forall t \in (t_{i,k}, t_{i,k+1})$

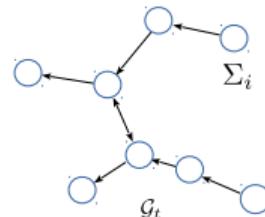
● Broadcast x_i when

$$|\hat{x}_i(t) - x_i(t)| \geq h_i(t)$$



Synchronization of multi-agent systems using event-triggered and self-triggered broadcasts

João Almeida, Carlos Silvestre, and António M. Pascoal

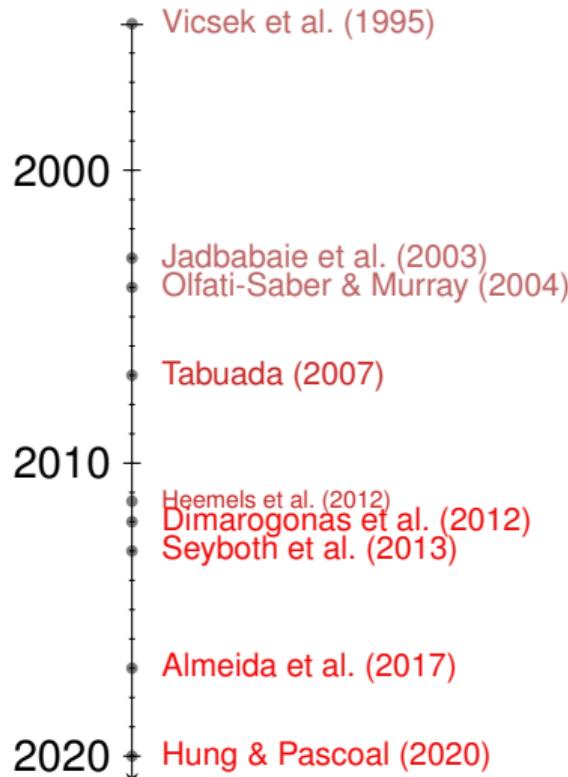


- ET-broadcast for synchronizing MAS

$$\Sigma_i : \dot{\mathbf{x}}_i = A\mathbf{x}_i + \mathbf{u}_i, \quad i = 1, \dots, N.$$

- ET-broadcast decides when agents broadcast their information (state).

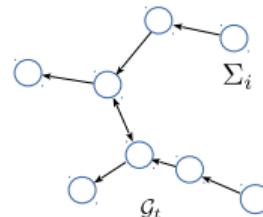
Cooperative control of MAS ▷ A brief review



International Journal of Control 2020, to appear

Consensus/synchronization of networked nonlinear multiple agent systems with event-triggered communications

Nguyen T. Hung^a and Antonio M. Pascoal^a

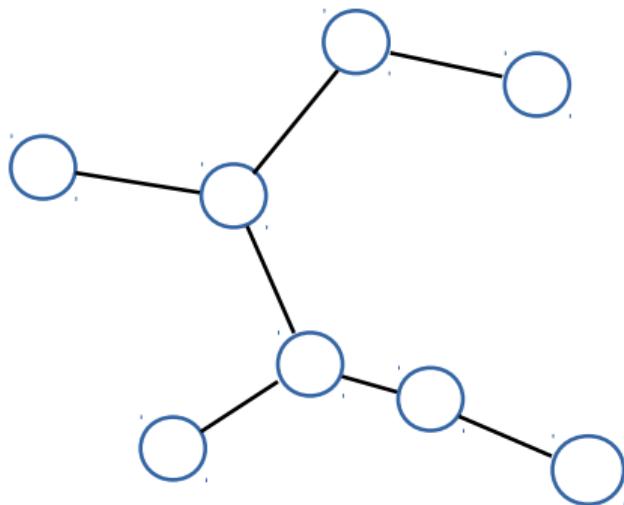


- ET-broadcast for synchronizing MAS

$$\Sigma_i : \dot{\mathbf{x}}_i = A\mathbf{x}_i + \mathbf{f}(\mathbf{x}_i, t) + B\mathbf{u}_i, \quad i = 1, \dots, N.$$

- ET-broadcast decides when agents broadcast their information (state).

Cooperative of MAS with event-triggered communications

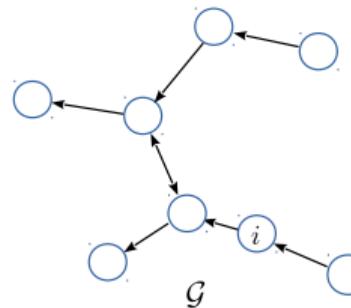


Consensus/synchronization of MAS ▷ Problem definition

A multi agent system with N agents:

$$\Sigma_i : \dot{\mathbf{x}}_i = A\mathbf{x}_i + \mathbf{f}(\mathbf{x}_i, t) + B\mathbf{u}_i, \quad i = 1, \dots, N,$$

Assm: $\mathbf{f}(\mathbf{x}, t)$ is Lipschitz in \mathbf{x} .



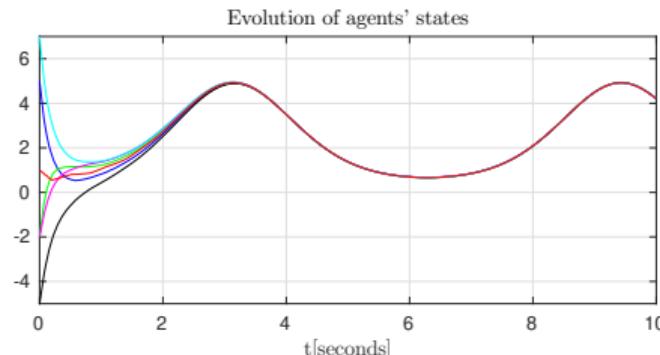
Problem (Consensus/synchronization)

Find $\mathbf{u}_i(\mathbf{x}_i, \mathbf{x}_j); j \in \mathcal{N}_i^{\text{in}a}$ s.t.

- $\lim_{t \rightarrow \infty} \mathbf{x}_i(t) - \mathbf{x}_j(t) = 0,$
- $\lim_{t \rightarrow \infty} \dot{\mathbf{x}}_i = A\mathbf{x}_i + \mathbf{f}(\mathbf{x}_i, t),$

$\forall i, j = 1, \dots, N.$

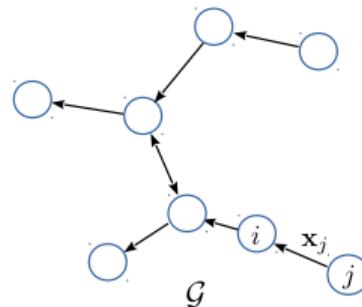
^aset of neighbors where i can receive information



Distributed control law:

$$\mathbf{u}_i = cK \sum_{j \in \mathcal{N}_i^{\text{in}}} \mathbf{x}_i - \mathbf{x}_j \quad (1)$$

↑
 $j \in \mathcal{N}_i^{\text{in}}$ ↑
From in-neighbors
Feedback gains



Result (Li et al. (2012))

The consensus/synchronization problem is solved with protocol (1) if

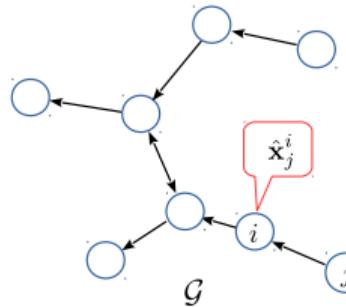
- c, K are chosen properly and
- \mathcal{G} is strongly connected.

Consensus/synchronization of MAS with ETC ▷ Concept

Distributed control law:

$$\mathbf{u}_i = cK \sum_{j \in \mathcal{N}_i^{\text{in}}} \mathbf{x}_i - \hat{\mathbf{x}}_j^i \quad (2)$$

↑
estimated neighbor's state



$$\hat{\mathbf{x}}_j^i : \begin{cases} \dot{\hat{\mathbf{x}}}_j^i = A\hat{\mathbf{x}}_j^i + \mathbf{f}(\hat{\mathbf{x}}_j^i, t) \\ \hat{\mathbf{x}}_j^i(t_{ij,k}^+) = \mathbf{x}_j(t_{j,k}) \end{cases} \quad (3)$$

Time instant at which
i receives Time instant at which
j broadcasts

$$(2) \Leftrightarrow$$

$$\mathbf{u}_i = cK \sum_{j \in \mathcal{N}_i^{\text{in}}} \mathbf{x}_i - \mathbf{x}_j + \mathbf{e}_j$$

$\mathbf{e}_j \triangleq \hat{\mathbf{x}}_j^i - \mathbf{x}_j$

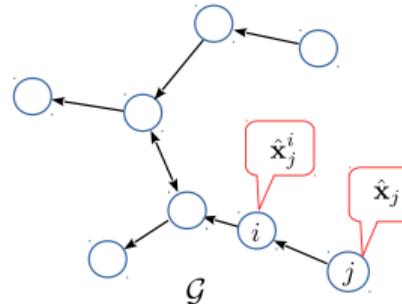
- \mathbf{e}_j : the estimation error of *j* at *i*.
- Our wish: If \mathbf{e}_j “big”, *j* must transmit the latest of \mathbf{x}_j to *i* to update $\hat{\mathbf{x}}_j^i$. But how?

Consensus/synchronization of MAS with ETC ▷ Concept

Distributed control law:

$$\mathbf{u}_i = cK \sum_{j \in \mathcal{N}_i^{\text{in}}} \mathbf{x}_i - \hat{\mathbf{x}}_j^i \quad (2)$$

↑
estimated neighbor's state



$$\dot{\hat{\mathbf{x}}}_j^i : \begin{cases} \dot{\hat{\mathbf{x}}}_j^i = A\hat{\mathbf{x}}_j^i + \mathbf{f}(\hat{\mathbf{x}}_j^i, t) \\ \hat{\mathbf{x}}_j^i(t_{ij,k}^+) = \mathbf{x}_j(t_{j,k}) \end{cases} \quad (3)$$

Time instant at which
 i receives

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 j broadcasts

$$\dot{\hat{\mathbf{x}}}_j : \begin{cases} \dot{\hat{\mathbf{x}}}_j = A\hat{\mathbf{x}}_j + \mathbf{f}(\hat{\mathbf{x}}_j, t) \\ \hat{\mathbf{x}}_j^i(t_{j,k}^+) = \mathbf{x}_j(t_{j,k}) \end{cases} \quad (4)$$

If there is no communication delay:

$$\hat{\mathbf{x}}_j^i(t) = \hat{\mathbf{x}}_j(t) \Rightarrow \mathbf{e}_j(t) = \mathbf{x}_j(t) - \hat{\mathbf{x}}_j(t) \quad \forall t.$$

Monitor $\mathbf{e}_j(t)$ and broadcast $\mathbf{x}_j(t)$ whenever

A designed threshold

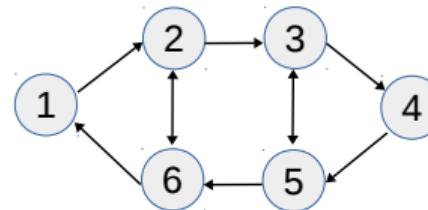
$$\|\mathbf{e}_j(t)\| \geq h_j(t).$$

Consensus/synchronization of MAS with ETC ▷ Illustrative examples

A multi agent system with 6 agents:

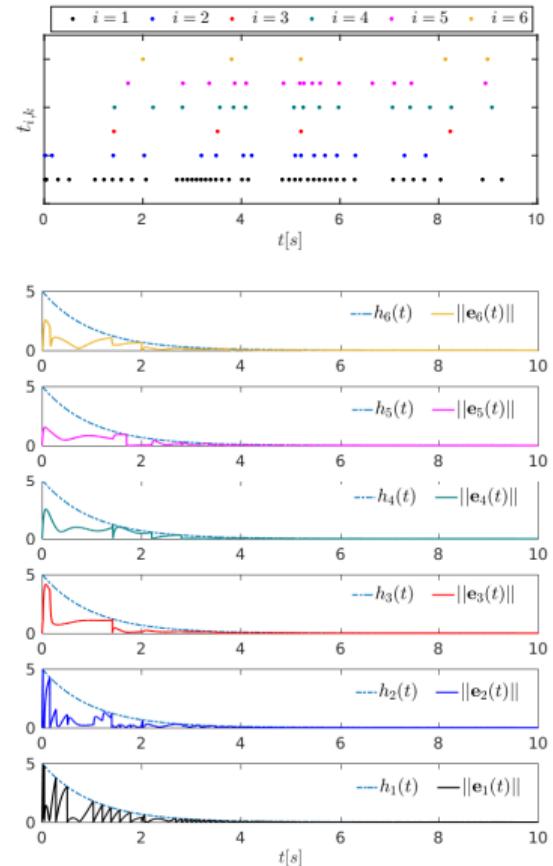
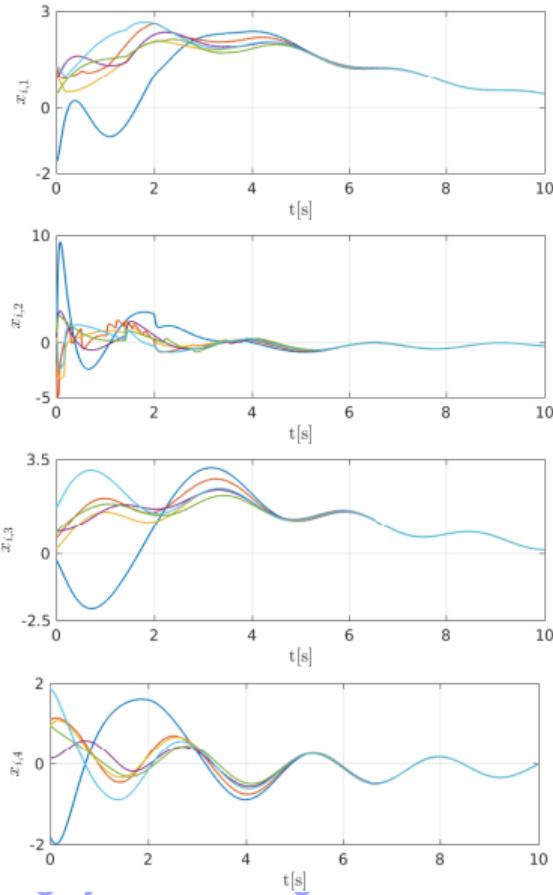
$$\Sigma_i : \dot{\mathbf{x}}_i = A\mathbf{x}_i + \mathbf{f}(\mathbf{x}_i, t) + B\mathbf{u}_i, \quad i = 1, \dots, N,$$

$$\mathbf{x}_i = [x_{i1}, x_{i2}, x_{i3}, x_{i4}]^T.$$



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -1 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \quad B = [0 \ 1 \ 0 \ 0]^T, \quad \mathbf{f}(\mathbf{x}_i, t) = [0 \ 0 \ 0 \ -0.333 \sin(x_{i3})]^T$$

Consensus/synchronization of MAS with ETC ▷ Illustrative examples



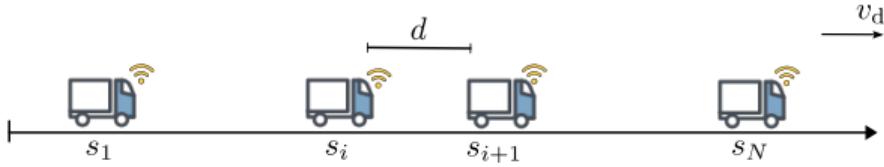
Applications in cooperative control of autonomous vehicles



Vehicle platooning

Vehicle model:

$$\dot{s}_i = v_i, \quad \forall i = 1, \dots, N \quad (5)$$



Change coordinate: For $\forall i = 1, N$

$$x_i \triangleq s_i - (i-1)d, \quad v_i \triangleq v_d(t) + u_i.$$

$$(5) \Leftrightarrow \dot{x}_i = v_d(t) + u_i, \quad \forall i = 1, \dots, N.$$

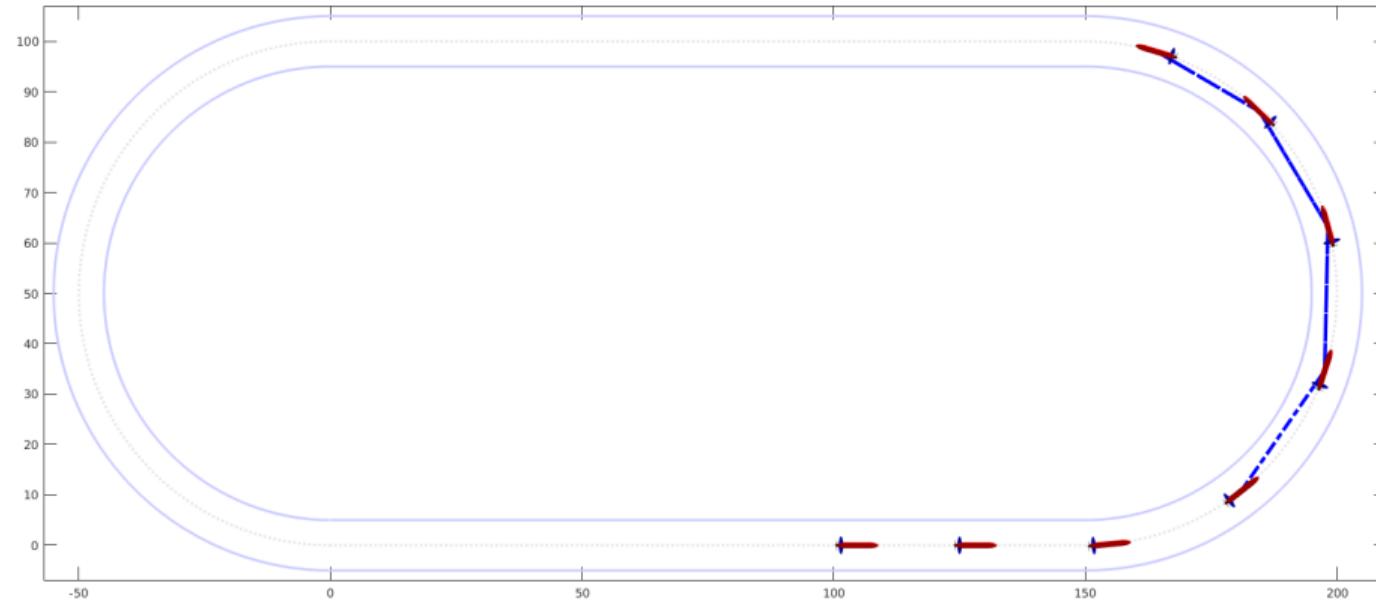
Problem (reformulated)

Find $u_i(x_i, x_j); j \in \mathcal{N}_i$ s.t.

- $\lim_{t \rightarrow \infty} x_i(t) - x_j(t) = 0,$
 $\forall i = 1, \dots, N.$
- $\lim_{t \rightarrow \infty} \dot{x}_i = v_d(t),$
 $\forall i = 1, \dots, N.$

^a $\mathcal{N}_i \triangleq \{i-1, i+1\}$

Vehicle platooning



Cooperative path following (CPF)

Problem formulation

Vehicle i – V_i : Rot. matrix $\{\mathcal{B}^{[i]}\}$ to $\{\mathcal{I}\}$

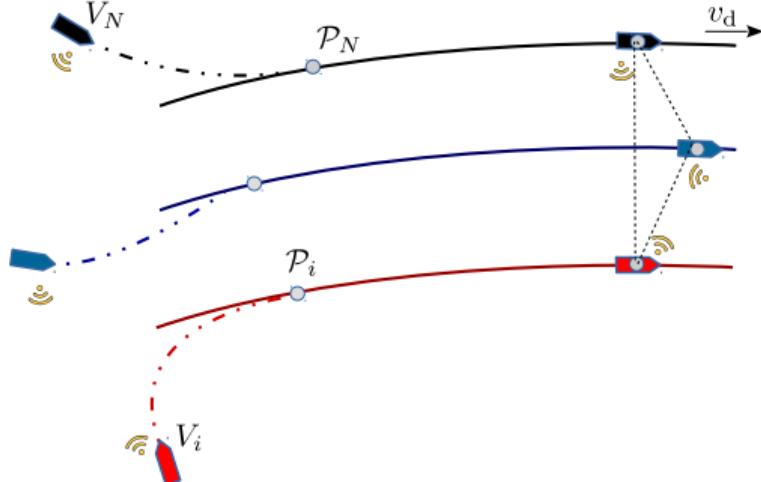
$$\begin{aligned}\dot{\mathbf{p}}^{[i]} &= R(\boldsymbol{\eta}^{[i]})\mathbf{v}^{[i]}, \\ \dot{\boldsymbol{\eta}}^{[i]} &= T(\boldsymbol{\eta}^{[i]})\boldsymbol{\omega}^{[i]}\end{aligned}$$

Orientation vector

Path i – \mathcal{P}_i :

$$\mathbf{p}_d^{[i]} : \gamma^{[i]} \rightarrow \mathbf{p}_d^{[i]}(\gamma^{[i]})$$

↑
path parameterizing variable
(e.g. path length)



Cooperative path following (CPF)

Problem formulation

Problem (CPF)

Find $\mathbf{v}^{[i]}, \omega^{[i]}, \dot{\gamma}^{[i]}$ to fulfill

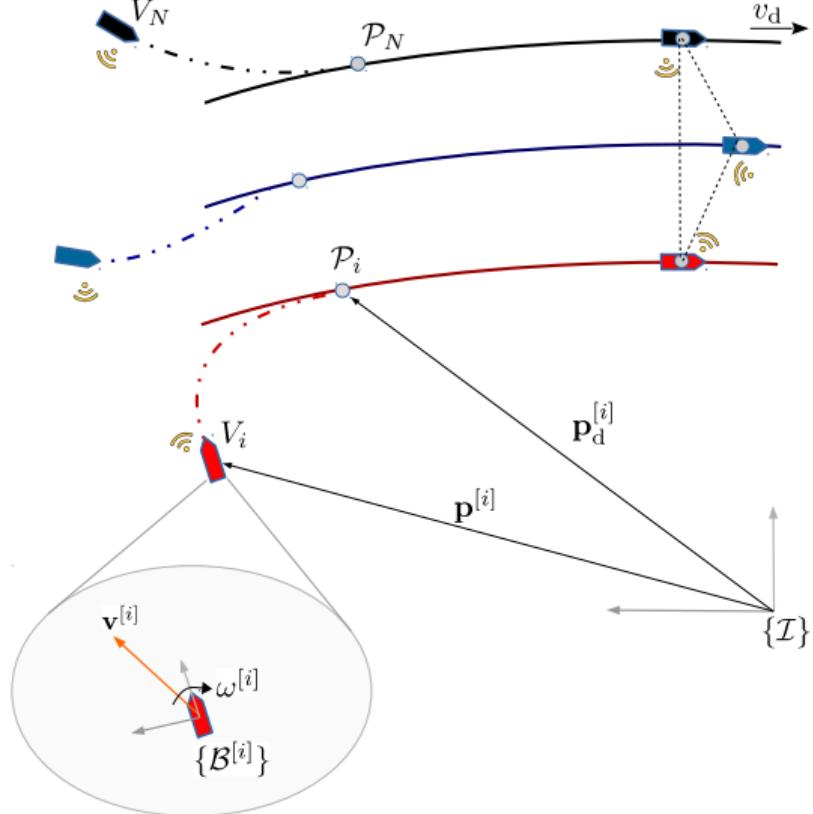
① Path following task:

- ▶ $\mathbf{p}^{[i]}(t) \rightarrow \mathbf{p}_d^{[i]}(\gamma^{[i]}(t))$

② Cooperative task:

- ▶ $\gamma^{[i]}(t) \rightarrow \gamma^{[j]}(t), \quad \forall i, j$
- ▶ $\dot{\gamma}^{[i]}(t) \rightarrow v_d, \quad \forall i$

as $t \rightarrow \infty$.



Cooperative path following (CPF)

Problem formulation

Problem (CPF)

Find $\mathbf{v}^{[i]}, \omega^{[i]}, \dot{\gamma}^{[i]}$ to fulfill

① Path following task:

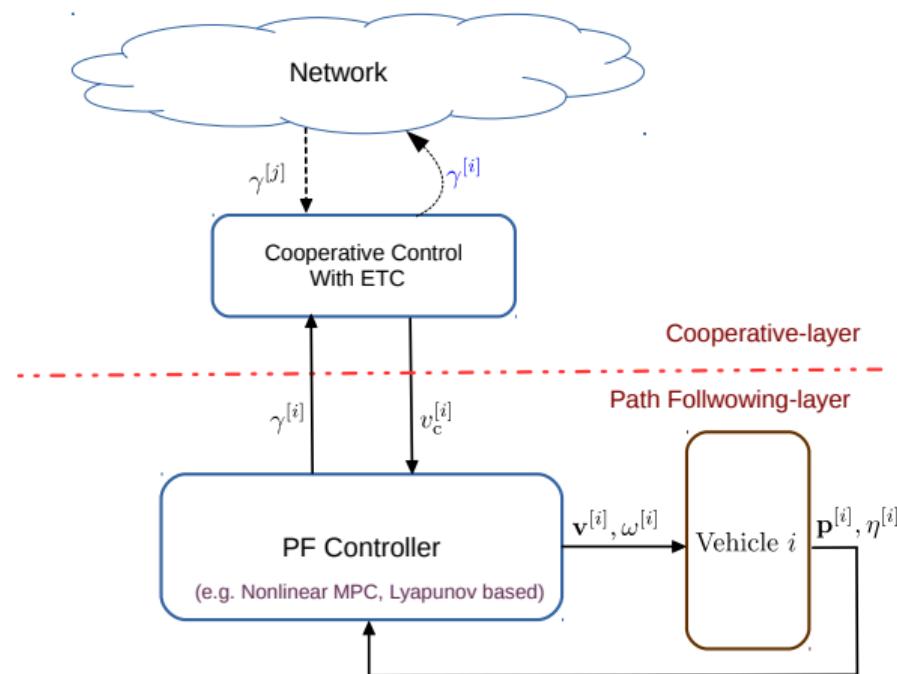
- ▶ $\mathbf{p}^{[i]}(t) \rightarrow \mathbf{p}_d^{[i]}(\gamma^{[i]}(t))$

② Cooperative task:

- ▶ $\gamma^{[i]}(t) \rightarrow \gamma^{[j]}(t), \quad \forall i, j$
- ▶ $\dot{\gamma}^{[i]}(t) \rightarrow v_d, \quad \forall i$

as $t \rightarrow \infty$.

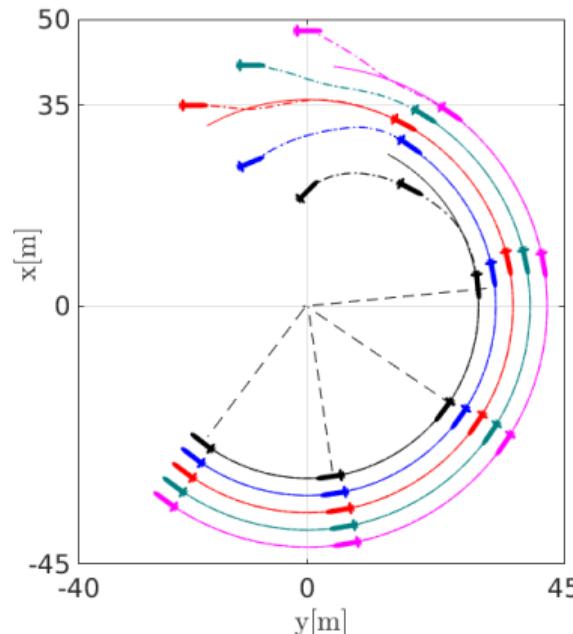
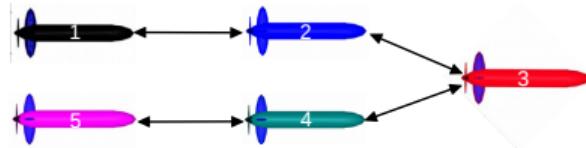
CPF control system for V_i ^a



CPF - simulation examples

Setup (Hung et al. (2020))

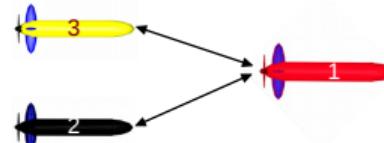
- 5 constrained under-actuated vehicles
- PF controller: Nonlinear MPC



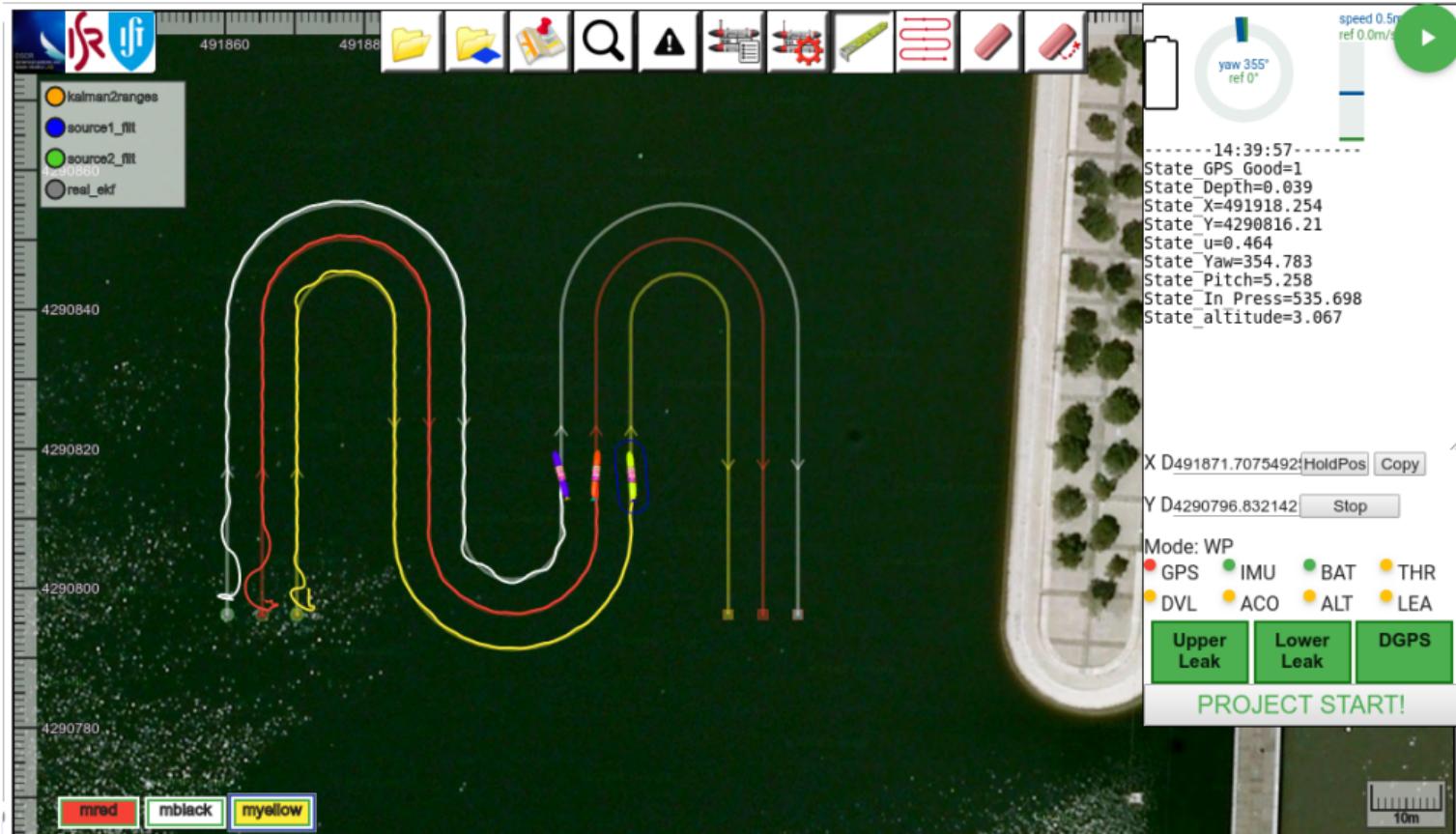
CPF - real tests

Setup (Rego et al. (2019))

- 3 Medusa class AUVs
- PF controller: Lyapunov based controller



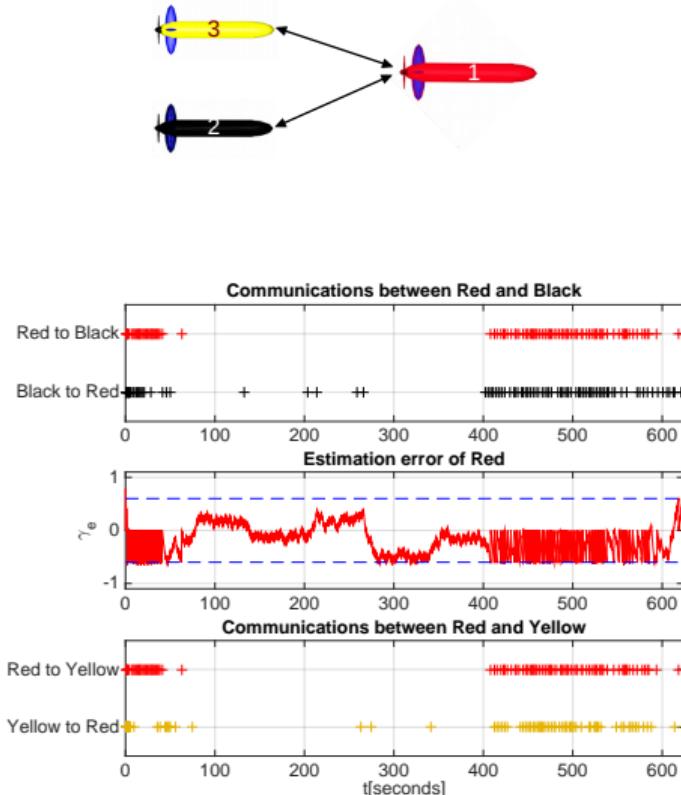
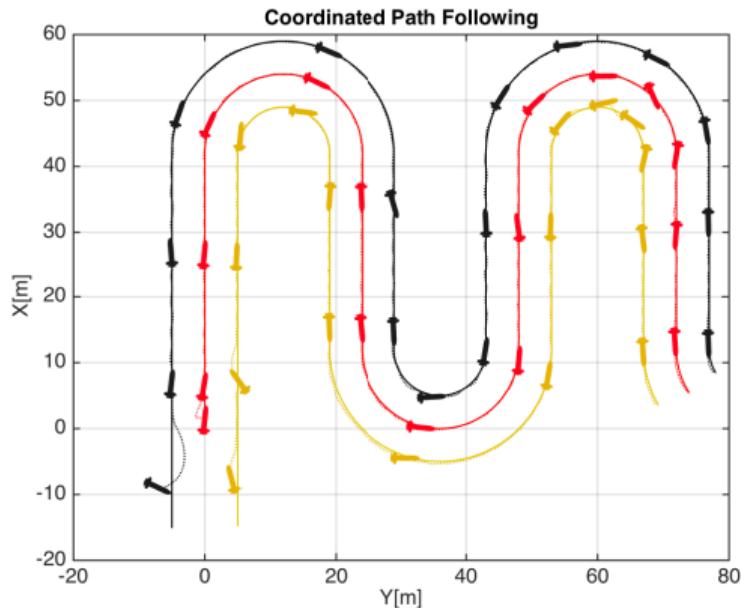
CPF - real tests



CPF - real tests

Setup Rego et al. (2019)

- 3 Medusa class AUVs
- PF controller: Lyapunov based controller



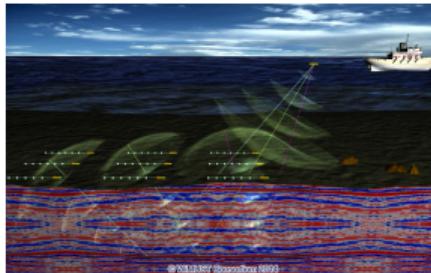
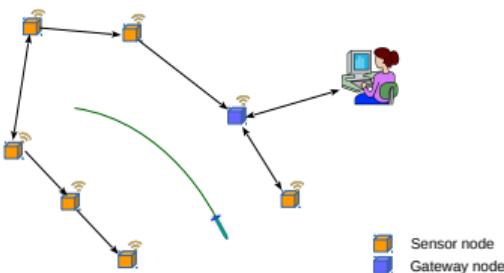
Others applications

Other applications

To learn (model) nature's behaviors



Engineering applications



Thank you!

Other videos:

Wimust project: [click here](#).

US navy: [click here](#).

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